

Supersymmetric curvatons and phase-induced curvaton fluctuations

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We consider the curvaton scenario in the context of supersymmetry (SUSY) with gravity-mediated SUSY breaking. In the case of a large initial curvaton amplitude during inflation and a negative order H^2 correction to the mass squared term after inflation, the curvaton will be close to the minimum of its potential at the end of inflation. In this case the curvaton amplitude fluctuations will be damped due to oscillations around the effective minimum of the curvaton potential, requiring a large expansion rate during inflation in order to account for the observed energy density perturbations, in conflict with cosmic microwave background constraints. Here we introduce a new curvaton scenario, the phase-induced curvaton scenario, in which de Sitter fluctuations of the phase of a complex SUSY curvaton field induce an amplitude fluctuation that is unsuppressed even in the presence of a negative order H^2 correction and large initial curvaton amplitude. This scenario is closely related to the Affleck-Dine mechanism and a curvaton asymmetry is naturally generated in conjunction with the energy density perturbations. Cosmological energy density perturbations can be explained with an expansion rate $H \approx 10^{12}$ GeV during inflation.

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I. INTRODUCTION

There has recently been considerable interest in the possibility that the energy density perturbations leading to structure formation could originate as quantum fluctuations of a scalar field other than the inflaton [1–3]. This scalar field has been called the curvaton. A number of candidates for the curvaton have been suggested and analyses performed [4–6]. The curvaton scenario effectively decouples the inflaton energy density from the observed cosmic microwave background (CMB) temperature fluctuations, allowing for a lower energy density during inflation. This could be advantageous in certain inflation models such as D -term hybrid inflation [7], which requires a curvaton in order to evade generation of unacceptable CMB fluctuations due to cosmic strings [8]. It also allows inflation to be driven by a scalar potential that would otherwise be ruled out by the scale dependence of its perturbation spectrum.

In supersymmetric (SUSY) models with gravity-mediated SUSY breaking [9,10], the curvaton is a complex field with a nontrivial scalar potential consisting of conventional soft SUSY breaking terms of the order of the weak scale, order H SUSY breaking corrections induced by energy densities in the early Universe [11–13], and nonrenormalizable superpotential terms suppressed by the natural mass scale of supergravity (SUGRA), $M = M_{Pl}/\sqrt{8\pi}$, where M_{Pl} is the Planck mass. The resulting SUSY curvaton evolution will be largely determined by the order H corrections to the curvaton potential.

In this paper we wish to show that there is an alternative curvaton scenario specific to gravity-mediated SUSY breaking, which we call the phase-induced curvaton scenario. In this scenario quantum fluctuations of the phase of the curvaton are transferred to amplitude fluctuations at the onset of

curvaton coherent oscillations. This scenario will be important in the case where there is a large curvaton amplitude during inflation together with a negative order H^2 curvaton mass squared term after inflation. As we will show, in this case the conventional curvaton scenario based on amplitude fluctuations is typically inconsistent with CMB constraints.

The possibility that the phase field of a SUSY curvaton could play a role in generating the energy density perturbations has been considered previously in the context of gauge-mediated SUSY breaking [14]. The scenario discussed here is quite different; the phase field potential does not directly provide the energy density fluctuations (the energy density of the phase field being completely negligible in models with gravity-mediated SUSY breaking), but induces a fluctuation in the amplitude field which then serves as the curvaton as usual. The final curvaton energy density perturbation in the phase-induced curvaton scenario is typically of the same magnitude as in the original curvaton scenario (which has an effectively massless curvaton up to the onset of coherent curvaton oscillations) [3], but with the value of the curvaton amplitude during inflation fixed by the curvaton potential. This fixes the value of H during inflation in terms of the observed energy density perturbation and the dimension of the dominant Planck-scale-suppressed nonrenormalizable curvaton superpotential term.

The phase-induced curvaton scenario is closely related to the Affleck-Dine mechanism [15] and we will show that a curvaton asymmetry is naturally generated in conjunction with the energy density perturbations.

The paper is organized as follows. In Sec. II we discuss the curvaton scenario in the context of SUSY models and the evolution of the curvaton for the case of a large initial amplitude during inflation. In Sec. III we discuss the phase-induced curvaton scenario. In Sec. IV we present our conclusions.

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II. FLAT DIRECTION SUSY CURVATON WITH A LARGE INITIAL AMPLITUDE

We consider the curvaton to be a SUSY flat direction field with no renormalizable superpotential terms.¹ The scalar potential then has the form

$$V = (m_s^2 + cH^2)|\Phi|^2 + (AW + \text{H.c.}) + \frac{\lambda^2 d^2 |\Phi|^{2(d-1)}}{M^{2(d-3)}}, \quad (1)$$

where we consider a curvaton superpotential

$$W = \frac{\lambda \Phi^d}{M^{d-3}}. \quad (2)$$

$m_s \approx 100$ GeV is the gravity-mediated soft SUSY breaking scalar mass term and cH^2 is the order H^2 correction originating from nonzero F terms due to energy densities in the early Universe [11–13]. The magnitude of the nonrenormalizable coupling λ is naturally in the range from 1 to $1/d!$, the latter value being expected if the coupling arises from integrating out heavy fields in a complete theory [16]. The A term consists of a gravity-mediated term plus an order H correction, $A = A_s + aH$, where $A_s \approx 100$ GeV [13].

The values of c and a depend upon the couplings of the inflaton superfield to the curvaton [13]. After inflation a value of $|c|$ of the order of 1 is the most likely [13]. We will concentrate on the case with $c \approx -1$, since a positive value of c will result in a highly damped curvaton amplitude at the onset of curvaton oscillations, making it difficult for the curvaton to dominate the energy density before it decays. During inflation the value of $|c|$ depends upon the inflation model. If inflation is driven by an F term, then the most likely value is $|c| \approx 1$, whereas if it is driven by a D term (as in D -term inflation [7]), then $|c| = 0$. The latter is favored in order to have a sufficiently flat inflaton potential. The value of a depends upon the coupling of the inflaton superfield to the curvaton. If there is no linear coupling of the inflaton to the curvaton in the Kähler potential, as may occur as a result of a discrete symmetry ($S \leftrightarrow -S$) or an R symmetry (commonly introduced to eliminate dangerous nonrenormalizable inflaton superpotential corrections in SUSY hybrid inflation models [7,16]), then $a = 0$ throughout [13]. This case will be fundamental to the phase-induced curvaton scenario to be discussed in the next section.

We first review the conditions for a scalar field to play the role of a curvaton. Let $\Phi = \phi e^{i\theta}/\sqrt{2}$ and consider $\theta = 0$ for now. A basic condition for the amplitude field ϕ to serve as a curvaton is that the Universe becomes matter dominated by curvaton oscillations before the era of nucleosynthesis. For the case $c \approx -1$ after inflation, the curvaton oscillations be-

gin once $m_s^2 \approx |c|H^2$. The largest natural value of ϕ at the onset of curvaton oscillations will correspond to ϕ approximately at the minimum of its potential when $|c|H^2 \gtrsim m_s^2$. The minimum of the potential and so initial amplitude can be estimated by neglecting the A terms in the curvaton potential, since the A terms are of the same order as the other terms when $|c|H^2 \approx m_s^2$. Minimizing

$$V(\phi) \approx \frac{cH^2}{2} \phi^2 + \frac{\lambda^2 d^2 \phi^{2(d-1)}}{2^{d-1} M^{2(d-3)}} \quad (3)$$

gives the minimum as a function of H and d for $|c|H^2 \gtrsim m_s^2$,

$$\phi_m \approx \left(\frac{|c| 2^{d-2}}{\lambda^2 d^2 (d-1)} \right)^{1/(2d-4)} (M^{2(d-3)} H^2)^{1/(2d-4)}. \quad (4)$$

Coherent oscillations about $\phi = 0$ begin once $H < H_{osc} = m_s/|c|$. Let the value of ϕ at the onset of oscillations be ϕ_{osc} . The energy density in the coherently oscillating curvaton is then

$$\rho_\phi \approx \left(\frac{a_{osc}}{a} \right)^3 \frac{1}{2} m_s^2 \phi_{osc}^2, \quad (5)$$

where a is the scale factor. We assume for now that the curvaton oscillations begin when the Universe is matter dominated by inflaton oscillations (we refer to this as inflaton matter domination, IMD). This is true for reheating temperatures less than the thermal gravitino upper bound, $T_R \lesssim 10^8$ GeV [17], since $H_R \lesssim 1$ GeV $\ll H_{osc} \approx m_s$. IMD continues until the Universe becomes radiation dominated at the reheating temperature T_R . In this case

$$\begin{aligned} \rho_\phi &\approx \left(\frac{a_{osc}}{a_R} \right)^3 \left(\frac{a_R}{a} \right)^3 \frac{1}{2} m_s^2 \phi_{osc}^2 \\ &\equiv \left(\frac{g(T)}{g(T_R)} \right) \left(\frac{T}{T_R} \right)^3 \left(\frac{H_R}{H_{osc}} \right)^2 \frac{1}{2} m_s^2 \phi_{osc}^2, \end{aligned} \quad (6)$$

where $g(T)$ is the number of degrees of freedom in thermal equilibrium and H_R is the expansion rate at T_R . The energy density in the oscillating curvaton becomes equal to the background radiation density, $\rho_{rad} = (\pi^2/30)g(T)T^4$, once

$$\phi_{osc}^2 \approx \frac{6M^2}{|c|} \left(\frac{T}{T_R} \right). \quad (7)$$

With $\phi_{osc} \approx \phi_m(H_{osc})$, the condition that the curvaton dominates the energy density before curvaton decay at T_d then requires that

$$\begin{aligned} k_d (M^{2(d-3)} m_s^2)^{1/(d-2)} &\gtrsim \frac{6M^2}{|c|} \left(\frac{T_d}{T_R} \right) \\ &\approx 3.5 \times 10^{26} \frac{1}{|c|} \left(\frac{T_d}{1 \text{ MeV}} \right) \\ &\times \left(\frac{10^8 \text{ GeV}}{T_R} \right) \text{ GeV}, \end{aligned} \quad (8)$$

¹An example is provided by a Dirac right-handed sneutrino, although this requires additional interactions in order to ensure no dangerous lightest SUSY particle (LSP) density from late curvaton decay [6] and to generate the baryon asymmetry after curvaton decay [5].

where

$$k_d = \left(\frac{2^{d-2}}{\lambda^2 d^2 (d-1)} \right)^{1/(d-2)}. \quad (9)$$

In this we have scaled T_d to the temperature at nucleosynthesis, $T_{nuc} \approx 1$ MeV, since the curvaton must decay before nucleosynthesis. For $d=4,5,6$ the left-hand side of Eq. (8) is

$$k_4 (M^2 m_s^2)^{1/2} = 2.4 \times 10^{20} k_4 \left(\frac{m_s}{100 \text{ GeV}} \right) \text{ GeV}, \quad (10)$$

$$k_5 (M^4 m_s^2)^{1/3} = 6.9 \times 10^{25} k_5 \left(\frac{m_s}{100 \text{ GeV}} \right)^{2/3} \text{ GeV}, \quad (11)$$

and

$$k_6 (M^6 m_s^2)^{1/4} = 3.7 \times 10^{28} k_6 \left(\frac{m_s}{100 \text{ GeV}} \right)^{1/2} \text{ GeV}. \quad (12)$$

The value of k_d is approximately in the range 1 to 10 for λ varying from 1 to $1/d!$. With $k_d \approx 1$ this implies that $d \geq 5$ must be satisfied (with $d=5$ marginal) in order to satisfy the curvaton condition if T_R satisfies the conventional thermal gravitino upper bound on the reheating temperature, $T_R \lesssim 10^8$ GeV [17].

It is possible that T_R could be larger than the conventional thermal gravitino upper bound if the Universe is dominated by the curvaton energy density for a sufficiently long period [14]. In this case the thermal gravitinos are diluted by entropy production. Once $T_R > T_* \approx 10^{10}$ GeV the curvaton oscillations will begin during radiation domination ($H_R > H_{osc} = m_s/|c|^{1/2}$). In this case it is straightforward to show that T_* replaces T_R on the right-hand side of Eq. (7). Therefore the lower bound Eq. (8) can be relaxed by at most a factor 10^{-2} . This allows $d=5$ to be more plausible but still rules out $d=4$.

The second fundamental condition to have a successful curvaton scenario is that the magnitude of the energy density perturbation should be consistent with CMB observations. This requires that $\delta_\rho = 2\delta_\phi \approx 10^{-5}$, where $\delta_\rho = \delta\rho/\rho$ is the energy density perturbation and $\delta_\phi = \delta\phi/\phi$ is the perturbation of the curvaton oscillation amplitude ($\rho \propto \phi^2$). The value of δ_ϕ depends upon the evolution of the curvaton after inflation. For $c \approx -1$ after inflation there are two possibilities:

(i) ϕ , initially small, is still evolving towards ϕ_m at the onset of coherent oscillations. (This possibility was discussed in Ref. [6].)

(ii) ϕ reaches ϕ_m before the onset of oscillations.

In case (ii) the curvaton amplitude perturbation at the onset of oscillations will depend upon when ϕ reaches ϕ_m . We will consider the case where ϕ is close to ϕ_m at the end of inflation. This will be the situation if $c \approx -1$ during inflation, as in F -term inflation, in which case the curvaton amplitude will be rapidly damped to $\phi_m(H_I)$. It will also be true in the case where $c \approx 0$ during inflation, as in D -term inflation, if at the onset of inflation the flat direction field has a value

greater than the value of the minimum at the end of inflation, $\phi_m(H_I)$. For example, this would be expected if the Universe evolves from chaotic initial conditions such that $V(\phi) \approx M^4$ initially [18,19]. It would also be expected if all values of ϕ at the onset of inflation were equally likely up to the value at which the curvaton effective mass becomes dynamically significant, $V''(\phi) \approx H_I^2$. In this case the average value of ϕ at the onset of inflation will satisfy $\phi \gtrsim \phi_*$, where ϕ_* denotes the value of ϕ at which $V''(\phi) \approx H_I^2$. (A similar argument in the context of axion cosmology has been given in Ref. [20].) If initially $\phi \gtrsim \phi_*$, then the curvaton will undergo rapid damped oscillations in a $\phi^{2(d-1)}$ potential until the amplitude of oscillation reaches ϕ_* , shortly after which the amplitude evolution will become highly damped and its value effectively frozen. ϕ_* is given by

$$\phi_* = \left(\frac{1}{|c|(2d-3)} \right)^{1/(2d-4)} \phi_m(H_I). \quad (13)$$

Thus if the initial curvaton amplitude is greater than or of the order of ϕ_* we expect $\phi \approx \phi_* = O(1)\phi_m(H_I)$ at the end of inflation. In the following we will assume that ϕ reaches ϕ_* before the length scales relevant to cosmological structure formation leave the horizon, since otherwise a highly scale-dependent curvaton perturbation spectrum would be obtained [19].

We next consider the evolution of a superhorizon spatial perturbation of the curvaton amplitude. Assuming the curvaton is effectively massless during inflation, the magnitude of the perturbation on a given scale at horizon crossing is $\delta\phi = H_I/2\pi$. The scalar field equation is

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2}{a^2}\phi = -V'(\phi), \quad (14)$$

where $\phi(\mathbf{x}, t) = \phi_o(t) + \delta\phi(\mathbf{x}, t)$. $\phi_o(t)$ is the homogeneous curvaton amplitude and $\delta\phi(\mathbf{x}, t)$ is the spatial fluctuation due to quantum fluctuations. For a fluctuation of wave number $k \ll H$, the gradient term in Eq. (14) is negligible and the evolution of the field at a point in space is determined by

$$\ddot{\phi} + 3H\dot{\phi} \approx -V'(\phi). \quad (15)$$

For $|c|H^2 \gg m_s^2$ and with $H \propto a^{-m}$, a solution of Eq. (15) is given by $\phi = \bar{\phi}_m \equiv K\phi_m$, where

$$K^{2d-4} = 1 + \frac{1}{|c|(d-2)} \left[3m - m^2 \left(\frac{d-1}{d-2} \right) \right]. \quad (16)$$

If the curvaton amplitude is initially at or close to $\bar{\phi}_m$ at the end of inflation, then the subsequent evolution of ϕ at a point in space will correspond to oscillations about $\bar{\phi}_m$. Substituting $\phi = \bar{\phi}_m + \Delta\phi$, where $\Delta\phi(\mathbf{x}, t) = \Delta\phi_o(t) + \delta\phi(\mathbf{x}, t) \ll \bar{\phi}_m$ and $\Delta\phi_o(t) = \phi_o - \bar{\phi}_m$, the equation for a superhorizon perturbation $\delta\phi(\mathbf{x}, t)$ at a point in space becomes

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} \approx -\alpha H^2 \delta\phi, \quad (17)$$

where

$$\alpha = |c|[(2d-3)K^{2d-4} - 1]. \quad (18)$$

[$\Delta\phi_o(t)$ satisfies the same equation.] The general solution of Eq. (17) is $\delta\phi \propto a^\gamma$, where

$$\gamma = \frac{1}{2}[-(3-m) + \sqrt{(3-m)^2 - 4\alpha}]. \quad (19)$$

Thus, with $\delta\phi(\mathbf{x}, t) = \delta\phi(t)\sin(\mathbf{k}\cdot\mathbf{x})$, the evolution of the amplitude of a superhorizon quantum fluctuation at a point in space is given by $\delta\phi(t) \propto a^\gamma$. We can then compute the suppression of $\delta\phi$ due to expansion from the end of inflation until the onset of coherent curvaton oscillations. We assume that coherent curvaton oscillations begin during IMD, such that $m=3/2$. Then $\gamma = \frac{1}{2}[-\frac{3}{2} + \sqrt{\frac{9}{4} - 4\alpha}]$. Since $\alpha \sim 1$, we expect an oscillating solution for $\delta\phi(t)$ (i.e., imaginary root in γ) with oscillation amplitude scaling as $\delta\phi \propto a^{-3/4}$.

If ϕ_o is initially close to $\bar{\phi}_m$, then since $\Delta\phi_o/\bar{\phi}_m$ decreases with time, the mean value of the curvaton amplitude will become increasingly close to the effective minimum, $\phi_o \approx \bar{\phi}_m \propto H^{1/(d-2)} \propto a^{-3/[2(d-2)]}$. Thus if $\phi_o \approx \bar{\phi}_m$ at the end of inflation (corresponding to scale factor a_e) we have

$$\left(\frac{\delta\phi}{\phi}\right)_{osc} \approx \left(\frac{a_e}{a_{osc}}\right)^{\frac{3}{4}[1-2/(d-2)]} \left(\frac{\delta\phi}{\phi}\right)_I, \quad (20)$$

where $(\delta\phi/\phi)_I$ is the value of the quantum fluctuation at the end of inflation. [($\delta\phi/\phi)_I$ is constant during inflation since ϕ is frozen for $\phi \leq \phi_*$.] Once curvaton coherent oscillations begin, δ_ϕ remains constant since $V(\phi) \propto \phi^2$ and both ϕ_o and $\delta\phi$ evolve in the same way ($\propto a^{-3/2}$). Since K is very close to 1 ($K=1.044$ for $d=6$, $m=3/2$ and $|c|=1$) we can assume $\bar{\phi}_m \approx \phi_m$. With $\phi \approx \phi_m(H_I)$ and $\delta\phi \approx H_I/2\pi$ when the curvaton perturbations leave the horizon, we obtain

$$\delta_\rho \approx 2 \left(\frac{\delta\phi}{\phi}\right)_{osc} \approx \left(\frac{H_{osc}}{H_I}\right)^{\frac{1}{2}[1-2/(d-2)]} \frac{H_I}{\pi\phi_m(H_I)}. \quad (21)$$

This fixes H_I in terms of δ_ρ and d . With $\phi_m(H_I) \approx k_d^{1/2} M^{(d-3)/(d-2)} H_I^{1/(d-2)}$, the expansion rate during inflation is given by

$$H_I \approx \pi^2 k_d M^{[2(d-3)]/(d-2)} \left(\frac{|c|^{1/2}}{m_s}\right)^{(d-4)/(d-2)} \delta_\rho^2. \quad (22)$$

For example, for $d=6$ this requires that

$$H_I \approx \frac{\pi^2 k_6 |c|^{1/4} \delta_\rho^2 M^{3/2}}{m_s^{1/2}} \approx 3.7 \times 10^{17} |c|^{1/4} k_6 \left(\frac{\delta_\rho}{10^{-5}}\right)^2 \left(\frac{100 \text{ GeV}}{m_s}\right)^{1/2} \text{ GeV}, \quad (23)$$

whilst for $d=5$

$$H_I \approx \frac{\pi^2 k_5 |c|^{1/6} \delta_\rho^2 M^{4/3}}{m_s^{1/3}} \approx 6.8 \times 10^{14} |c|^{1/6} k_5 \left(\frac{\delta_\rho}{10^{-5}}\right)^2 \left(\frac{100 \text{ GeV}}{m_s}\right)^{1/3} \text{ GeV}, \quad (24)$$

where $k_5, k_6 \gtrsim 1$. These values of H_I correspond to an energy density that would result in unacceptably large energy density perturbations from conventional inflaton quantum fluctuations, which typically require $H_I \lesssim 10^{13}$ GeV in order to be compatible with CMB constraints. Thus in the case of a large curvaton amplitude during inflation and negative order H^2 correction after inflation, the energy density perturbations due to curvaton amplitude fluctuations are typically inconsistent with CMB constraints.

In the above we have assumed that quantum de Sitter fluctuations of the curvaton field during inflation are unsuppressed, i.e., that the curvaton effective mass is much smaller than H_I . In the case of inflation driven by an F term we expect that $c \approx -1$ for the flat direction field during inflation. In this case there will be effectively no superhorizon curvaton amplitude fluctuations on the scale of large scale structure formation, as these will be highly damped by curvaton evolution due to the order H^2 term during inflation. Thus a curvaton scenario based on fluctuations of the curvaton amplitude is ruled out in this case also.

However, as we will show in the next section, in both the large initial curvaton amplitude and F -term inflation cases it is still possible for a complex SUSY curvaton with gravity-mediated SUSY breaking to generate an energy density perturbation that is consistent with CMB constraints, via fluctuations of the curvaton phase.

III. PHASE-INDUCED CURVATON SCENARIO

If the inflaton has no linear coupling to the curvaton in the Kähler potential, then there will be no order H correction to the A terms [13,14,16]. This allows for a new version of the curvaton scenario that applies specifically to a complex SUSY curvaton with gravity-mediated SUSY breaking.

In the absence of order H corrections to the A terms, the phase field of the curvaton is effectively massless during and after inflation, even in the case of F -term inflation where the amplitude field typically gains an effective mass of order H . Thus de Sitter fluctuations of the phase will be unsuppressed. For ϕ constant during inflation, the canonically normalized phase field for small $\delta\theta$ about an amplitude in the θ direction

is $\phi_p \approx \phi_I \delta\theta$, where ϕ_I is the curvaton amplitude during inflation. Thus with $\delta\phi_p = H_I/2\pi$ the de Sitter fluctuation of the phase field is $\delta\theta \approx H_I/2\pi \phi_I$.

The equation of motion of the complex curvaton field is

$$\ddot{\Phi} + 3H\dot{\Phi} - \frac{\nabla^2}{a^2}\Phi = -\frac{\partial V}{\partial \Phi^\dagger} \equiv -\frac{e^{i\theta}\phi}{\sqrt{2}} \left[(m_s^2 + cH^2) + \frac{A\lambda d\phi^{d-2}}{M^{d-3}(\sqrt{2})^{d-2}} [\cos(d\theta) - i\sin(d\theta)] + \frac{\lambda^2 d^2 (d-1) \phi^{2(d-2)}}{2^{d-2} M^{2(d-3)}} \right]. \quad (25)$$

For perturbations of wavelength much larger than the horizon the gradient terms are negligible. In this case the evolution of the curvaton amplitude and phase at a point in space is given by

$$\ddot{\phi} + 3H\dot{\phi} - \dot{\theta}^2 \phi = - \left[(m_s^2 + cH^2) \phi + \frac{A\lambda d\phi^{d-1} \cos(d\theta)}{(\sqrt{2})^{d-2} M^{d-3}} + \frac{(d-1)\lambda^2 d^2 \phi^{2d-3}}{2^{d-2} M^{2(d-3)}} \right] \quad (26)$$

and

$$\ddot{\theta} + 3H\dot{\theta} + \frac{2\dot{\phi}}{\phi}\dot{\theta} = \frac{A\lambda d\phi^{d-2} \sin(d\theta)}{(\sqrt{2})^{d-2} M^{d-3}}. \quad (27)$$

From Eq. (27) we see that without an A term the phase at a point in space will remain constant. In the absence of order H corrections to the A term the phase will begin to evolve only when $m_s^2 \approx |c|H^2$, when the A term contribution to the potential becomes comparable with that of the mass squared term. A perturbation in the phase field will then induce a perturbation in the curvaton amplitude via the coupling proportional to $\cos(d\theta)$ in Eq. (26).

In our numerical results we focus on the case $d=6$ as an example. We assume that $c=-1$ and $\lambda=1$ throughout. (We use units such that $M=1$.) We evolve the equations of motion from a time corresponding to initial expansion rate $H^2 = 8000m_s^2 \gg |c|H^2$, with initial values $\phi = \bar{\phi}_m$, $\dot{\phi} = \dot{\bar{\phi}}_m$, $d\theta = \pi/4$, and $\dot{\theta}=0$. We set $a=1$ at this initial time. For the perturbations we assume $\delta\phi=0$ and $\delta\theta=10^{-5}$ initially. To calculate the evolution of the perturbation we evolve Eq. (26) and Eq. (27) for two points in space, one with initial value θ and one with initial value $\theta + \delta\theta$, corresponding to points in space with the mean value of θ and the largest perturbed value of θ , respectively.

In Fig. 1 we show the evolution of the energy density perturbation with scale factor for the case $A/m_s=1$. The spike feature is associated with the vanishing of the curvaton mass squared term at scale factor $a_{osc} (=19.44$ in our units) corresponding to $|c|H^2 = m_s^2$. At around this time the energy density briefly vanishes whilst changing sign, resulting in a

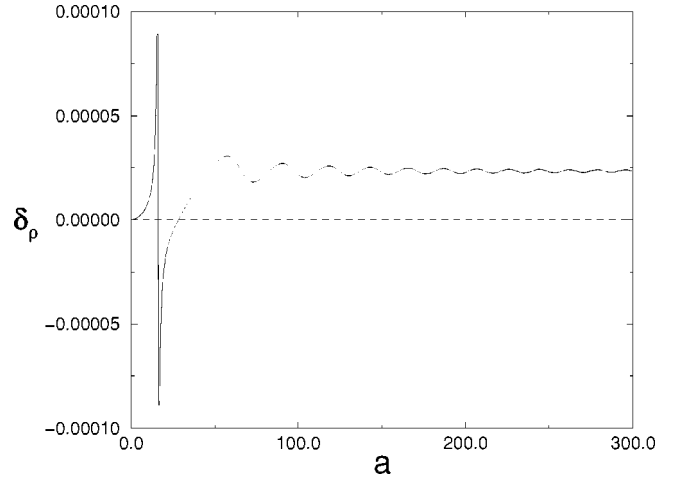


FIG. 1. Evolution of the energy density perturbation for $d=6$ and $A/m_s=1$.

large value for $\delta\rho/\rho$. Once a is large compared with a_{osc} , the nonrenormalizable terms in the curvaton potential become negligible as a result of the reduction of the curvaton amplitude due to expansion. Curvaton evolution in a $|\Phi|^2$ potential is then established with a constant curvaton energy density perturbation, numerically given by $\delta\rho = 2.3 \times 10^{-5}$ for the case $A/m_s=1$ and $\delta\theta=10^{-5}$.

The phase-induced curvaton scenario is closely related to the Affleck-Dine (AD) mechanism for the generation of global charge asymmetries such as baryon number [15]. The explicit dependence of the A term on the phase of the scalar field introduces CP violation into the equations of motion. The result is that the real and imaginary parts of Φ coherently oscillate out of phase with each other in a $|\Phi|^2$ potential once $a \gg a_{osc}$, such that the complex curvaton field describes an ellipse in the complex plane. Once $a > a_{osc}$ and the nonrenormalizable corrections become negligible, the curvaton potential has an approximately conserved global $U(1)$ symmetry, with respect to which a constant curvaton asymmetry in a comoving volume is established. The comoving curvaton charge asymmetry density is given by

$$n_\phi = a^3 i (\Phi^\dagger \dot{\Phi} - \dot{\Phi}^\dagger \Phi) \equiv a^3 \dot{\theta} \phi^2, \quad (28)$$

where the factor a^3 compensates for the expansion of the Universe, such that n_ϕ is constant for a conserved charge. In Fig. 2 we show the growth of the comoving curvaton asymmetry for the case $A/m_s=1$. This shows that a curvaton charge asymmetry naturally arises in conjunction with the energy density perturbations in the phase-induced curvaton scenario. θ introduces CP violation resulting in a curvaton asymmetry whilst $\delta\theta$ induces the energy density perturbations.

In Fig. 3 we show the evolution of the curvaton amplitude field in the form $a^{3/2}\phi$, where $a^{3/2}$ cancels out the effect of expansion on evolution in a $|\Phi|^2$ potential. For $a \gg a_{osc}$ we see that the curvaton amplitude has a maximum value ≈ 0.008 and a minimum value ≈ 0.002 , corresponding to the major and minor axes of the ellipse described by the globally charged curvaton field in the complex plane. In Fig. 4 we

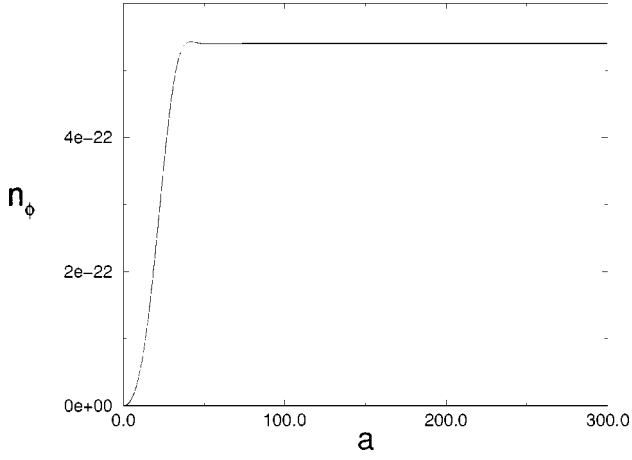


FIG. 2. Evolution of the comoving curvaton asymmetry for $d = 6$ and $A/m_s = 1$.

show the late time trajectory of the curvaton in the complex plane, where $\Phi = (\phi_1 + i\phi_2)/\sqrt{2}$.

In Table I we give the asymptotic values of δ_ρ , n_ϕ , and $\delta n_\phi/n_\phi$ (the spatial perturbation of the curvaton asymmetry) at $a \gg a_{osc}$ for different values of A/m_s . (n_ϕ gives the charge asymmetry at $a = 300$.) From this we see that the values of these quantities are sensitive to A/m_s . For $|A/m_s| = O(1)$ we broadly find that $|\delta_\rho| = O(1)|\delta_\theta|$ for $A > 0$ and $|\delta_\rho| = O(0.1)|\delta_\theta|$ for $A < 0$. The results in Table I are for the case $\lambda = 1$. We find that the values of δ_ρ and $\delta n_\phi/n_\phi$ are independent of the nonrenormalizable coupling λ .

In practice, the curvaton asymmetry is unlikely to result in any observable consequences. The curvaton asymmetry cannot produce the observed baryon asymmetry, since it has been shown that the decay of a complex Affleck-Dine curvaton condensate to a baryon asymmetry will result in large baryonic isocurvature fluctuations that are inconsistent with observations of the cosmic microwave background radiation [21].

From our numerical results we conclude that perturbations of the phase of the complex curvaton field can induce

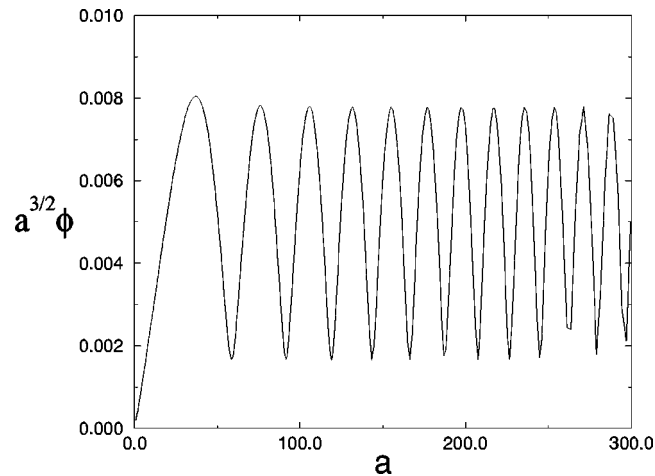


FIG. 3. Evolution of the curvaton amplitude for $d = 6$ and $A/m_s = 1$.

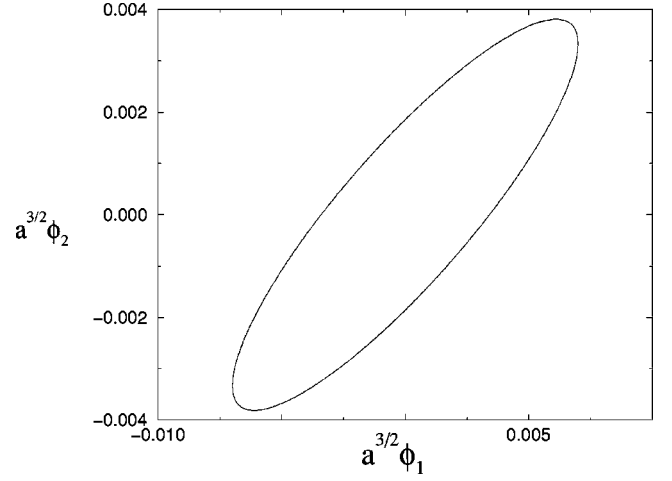


FIG. 4. Late-time trajectory of the curvaton in the complex plane.

an energy density perturbation $\delta_\rho = f_\theta \delta\theta$ with $|f_\theta|$ of the order of 1. This mechanism requires only that the A terms receive no order H corrections during or after inflation.

We note that the curvaton phase field itself does not significantly contribute to the energy density once $a \gg a_{osc}$. The A term in the scalar potential, responsible for the potential energy of the curvaton phase field, is proportional to ϕ^d . It is therefore completely negligible compared with the mass squared term once $a \gg a_{osc}$. For this reason it is essential that in gravity-mediated SUSY breaking the fluctuation of the curvaton phase is transferred to a fluctuation of the curvaton amplitude, in order to generate a significant energy density perturbation once $a \gg a_{osc}$. This is in contrast with the case of gauge-mediated SUSY breaking, where it is possible for the phase field potential to directly provide the curvaton fluctuation [14].

The energy density perturbation from the phase-induced curvaton scenario with a large initial amplitude is of the same order of magnitude as that obtained in the original curvaton scenario [3] for the case $\phi \approx \phi_m(H_I)$,

$$\delta_\rho = f_\theta \delta\theta \approx \frac{f_\theta H_I}{2\pi \phi_m(H_I)}. \quad (29)$$

TABLE I. Asymptotic values of δ_ρ , n_ϕ , and $\delta n_\phi/n_\phi$ as a function of A for the case $d = 6$ and $\delta\theta = 10^{-5}$.

A/m_s	δ_ρ	n_ϕ	$\delta n_\phi/n_\phi$
3.0	-1.4×10^{-5}	-1.9×10^{-22}	-5.0×10^{-4}
2.5	3.2×10^{-6}	-7.1×10^{-22}	-3.2×10^{-6}
2.0	1.7×10^{-5}	2.6×10^{-24}	-3.5×10^{-2}
1.5	2.6×10^{-5}	5.3×10^{-22}	-8.7×10^{-5}
1.0	2.3×10^{-5}	5.4×10^{-22}	6.0×10^{-7}
0.5	9.7×10^{-6}	2.8×10^{-22}	3.8×10^{-5}
-1	1.1×10^{-6}	-1.3×10^{-22}	9.0×10^{-5}
-2	5.5×10^{-7}	1.6×10^{-22}	7.5×10^{-5}
-3	-1.8×10^{-6}	-1.5×10^{-22}	6.1×10^{-5}

In the phase-induced curvaton scenario the value of H_I is fixed by δ_ρ and d , since $\phi_m(H_I)$ is determined by H_I and d . This is in contrast to the original curvaton scenario, where the value of ϕ during inflation can take any value [3]. The expansion rate in the phase-induced curvaton scenario is then given by

$$H_I \approx \left(\frac{2\pi k_d^{1/2} \delta_\rho}{f_\theta} \right)^{(d-2)/(d-3)} M. \quad (30)$$

An important feature of this expression is that H_I has an upper bound, corresponding to $d \rightarrow \infty$,

$$H_I \lesssim \frac{2\pi \delta_\rho M}{f_\theta} \approx 1.5 \times 10^{14} \frac{1}{f_\theta} \left(\frac{\delta_\rho}{10^{-5}} \right) \text{ GeV}, \quad (31)$$

where we have used $k_\infty = 1$. For smaller d the value of H_I is smaller. For example, for $d=5$ we obtain

$$H_I = \left(\frac{2\pi k_5^{1/2} \delta_\rho}{f_\theta} \right)^{3/2} M \approx 1.2 \times 10^{12} \left(\frac{k_5^{1/2}}{f_\theta} \right)^{3/2} \left(\frac{\delta_\rho}{10^{-5}} \right)^{3/2} \text{ GeV}, \quad (32)$$

whilst for $d=6$

$$H_I = \left(\frac{2\pi k_6^{1/2} \delta_\rho}{f_\theta} \right)^{4/3} M \approx 6.0 \times 10^{12} \left(\frac{k_6^{1/2}}{f_\theta} \right)^{4/3} \left(\frac{\delta_\rho}{10^{-5}} \right)^{4/3} \text{ GeV}. \quad (33)$$

For the $d=6$ example of Table I we have $f_\theta = 2.3$ for $A = m_s$, which implies that $H_I \approx 2 \times 10^{12}$ GeV for $k_6 \approx 1$.

Thus the values of H_I in the phase-induced curvaton scenario are typically less than or of the order of the values that are obtained in inflation models where the energy density perturbations are explained by conventional adiabatic inflation fluctuations. Therefore a complex SUSY curvaton can

consistently account for the observed energy density perturbations in the case of a large initial curvaton amplitude or F -term inflation, where the conventional SUSY curvaton scenario with amplitude fluctuations would be ruled out by CMB constraints.

IV. CONCLUSIONS

We have shown that it is possible for energy density perturbations in the SUSY curvaton scenario with gravity-mediated SUSY breaking to originate from quantum fluctuations of the phase of the complex curvaton field. The phase fluctuations induce curvaton amplitude fluctuations through the SUSY breaking A terms in the curvaton scalar potential. This requires that there are no order H corrections to the A term during and after inflation, which is true if the inflaton has no linear couplings in the Kähler potential. This can easily occur via a discrete symmetry or R symmetry, as is necessary in SUSY hybrid inflation models.

The phase-induced curvaton scenario becomes important when (i) the initial amplitude of the curvaton during inflation is large and (ii) after inflation there is a negative order H^2 correction to the curvaton mass squared term. Both of these are natural possibilities. In this case the conventional curvaton scenario based on amplitude fluctuations is likely to be inconsistent with cosmic microwave background constraints. The phase-induced curvaton scenario, on the other hand, can consistently account for the observed cosmological energy density perturbations even if (i) and (ii) are satisfied.

The phase-induced curvaton scenario is closely related to the Affleck-Dine mechanism and a curvaton asymmetry naturally occurs in conjunction with the energy density perturbations. However, it is unlikely that the curvaton asymmetry could result in observable consequences, since if the baryon asymmetry originates from a complex curvaton condensate via the Affleck-Dine mechanism, there would be unacceptably large baryon isocurvature perturbations [21].

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